

The Search for Exoplanets

Wednesday, 21 October 2020 9:57 am

2019, two thirds of the Physics Nobel Prize given to Michel Mayor and Didier Queloz were awarded the prize for finding the first exoplanet to be discovered orbiting a star like our own.

Since then, more than 4,000 planets have been discovered, of various sizes and using various methods.

Different methods to find Exoplanets

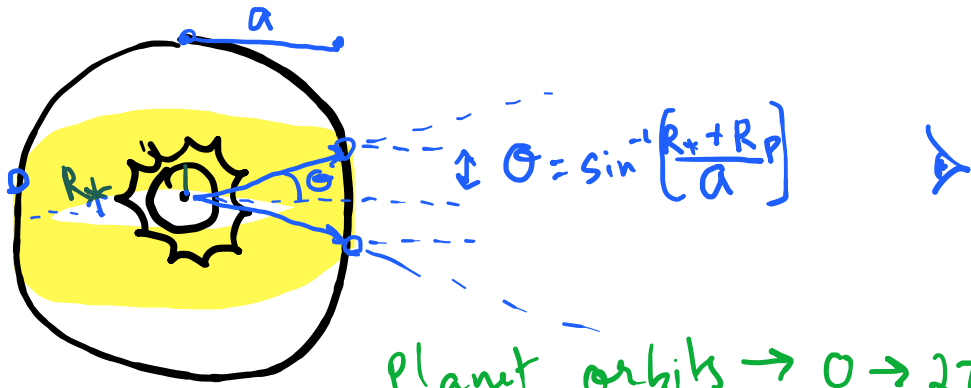
(1) Radial velocity: The gravitational pull exerted by a planet on a star leads to small shifts in the stellar light spectrum because of the Doppler effect. Of course these are very small shifts and this method also depends for some success on the orientation of the planetary orbit being in a strip reasonably close to our line of sight.

(2) Transit: As a planet crosses the stellar disc, there are periodic dips in the light intensity. Again, the orientation of the planetary orbit is crucial.

(3) Microlensing: If the light from an even more distant star can be observed as it passes very close to our candidate star, then the latter acts as a gravitational lens to enhance the light intensity received by us at Earth. Any planet can then act as a secondary lens, leading to fluctuations that can be observed. Such a lining up occurs only with very low probability, but extensive automatic surveys can lead to success with the technique.

(4) Direct: The faint image of a planet in a close orbit around an observed star can itself be observable directly, although there are problems with separation and contrast.

Probability of viewing an eclipsing planet



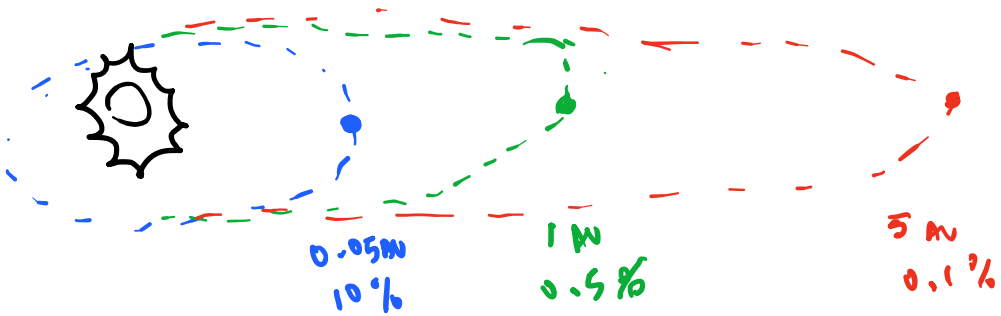
$$\therefore P(\text{transit}) = \frac{\int_{\theta_{\min}}^{\theta_{\max}} \int_0^{2\pi} \cos\theta \, d\theta \, d\phi}{\text{Total } \Omega \rightarrow 4\pi}$$

$$\Rightarrow P(\text{transit}) = \frac{R_* + R_p}{a}$$

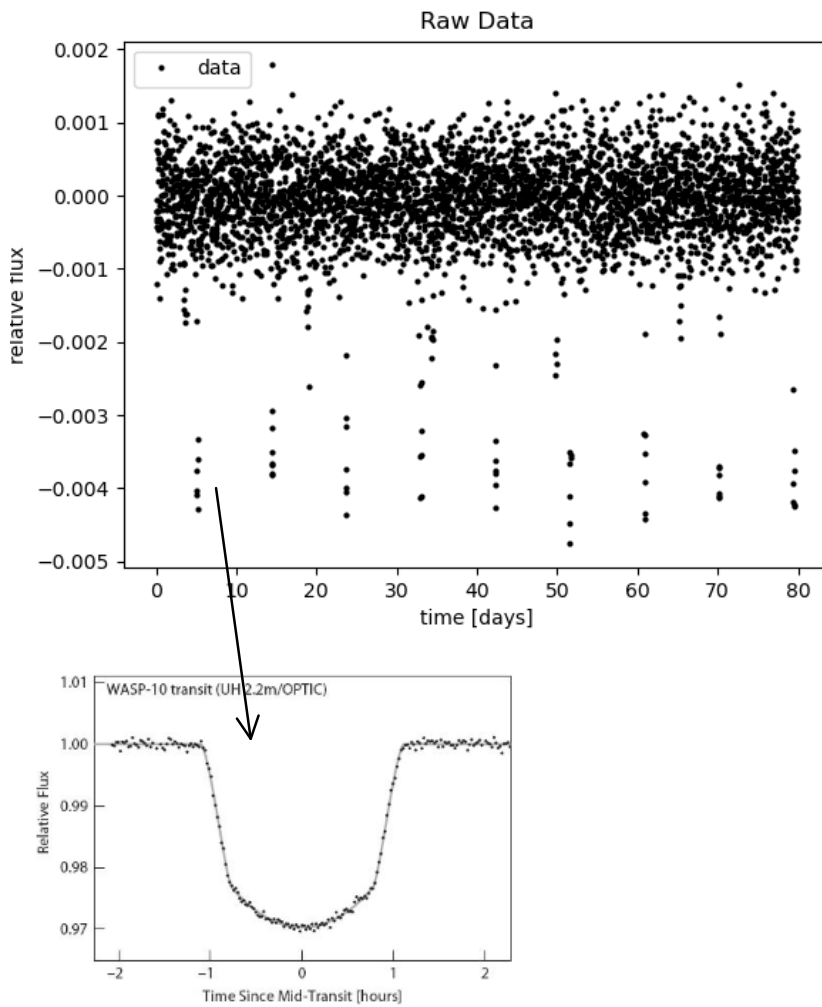
Eg for Earth-like $\rightarrow \frac{R_* + R_p}{a} \approx \frac{R_{\oplus}}{1 \text{ AU}} \sim 0.5\%$

Not Jupiter \rightarrow typical $T = 3 \text{ days}$
 $\Rightarrow a = T^{2/3}$
 $\Rightarrow a \sim 25$

$$\therefore P(\text{transit}) \sim 10\%$$



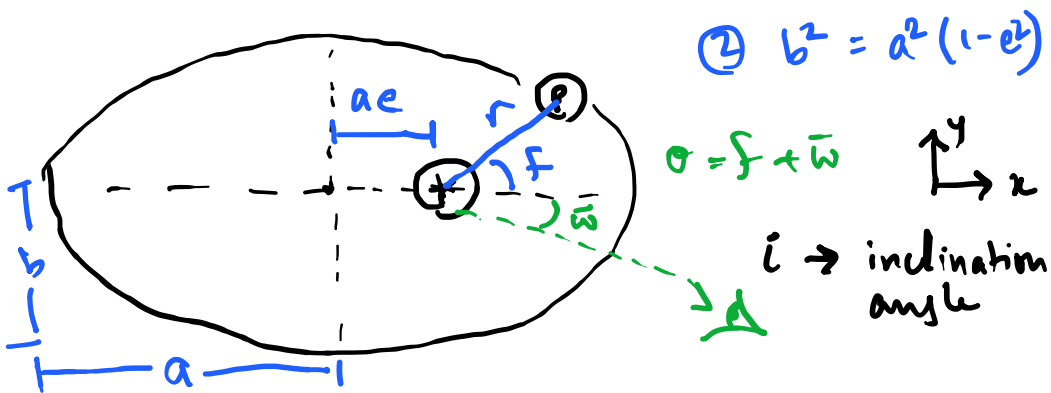
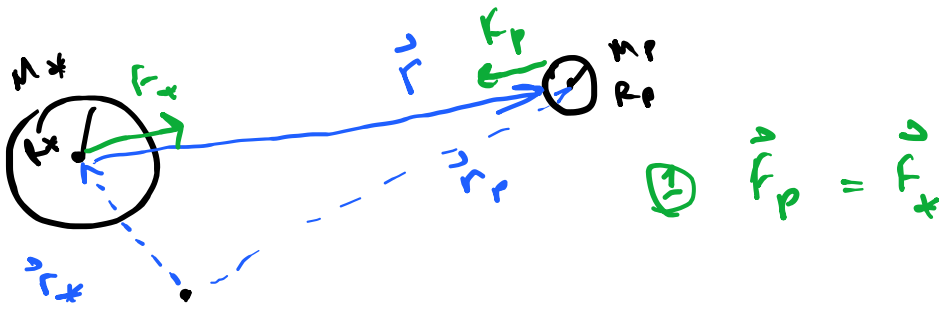
Light Curve Data



Physics that can be used to describe transit light curves:

1. Keplerian Orbit
2. Limb Darkening
3. Variability of Flux (noise)

1. Keplerian Orbit



Together $\rightarrow r = \frac{a(1-e^2)}{1+e \cos f}$

$\rightarrow x = -r \cos(\omega + f)$
 $y = -r \sin(\omega + f) \cos i$
 $z = r \sin(\omega + f) \sin i$

Minimize for eclipse \downarrow Kipping 2008 "Transiting planets"

$f_{\text{eclipse}} = \frac{\pi}{2} - \omega$

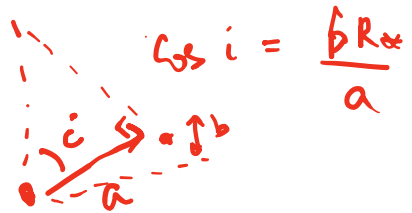
Finally

Impact parameter

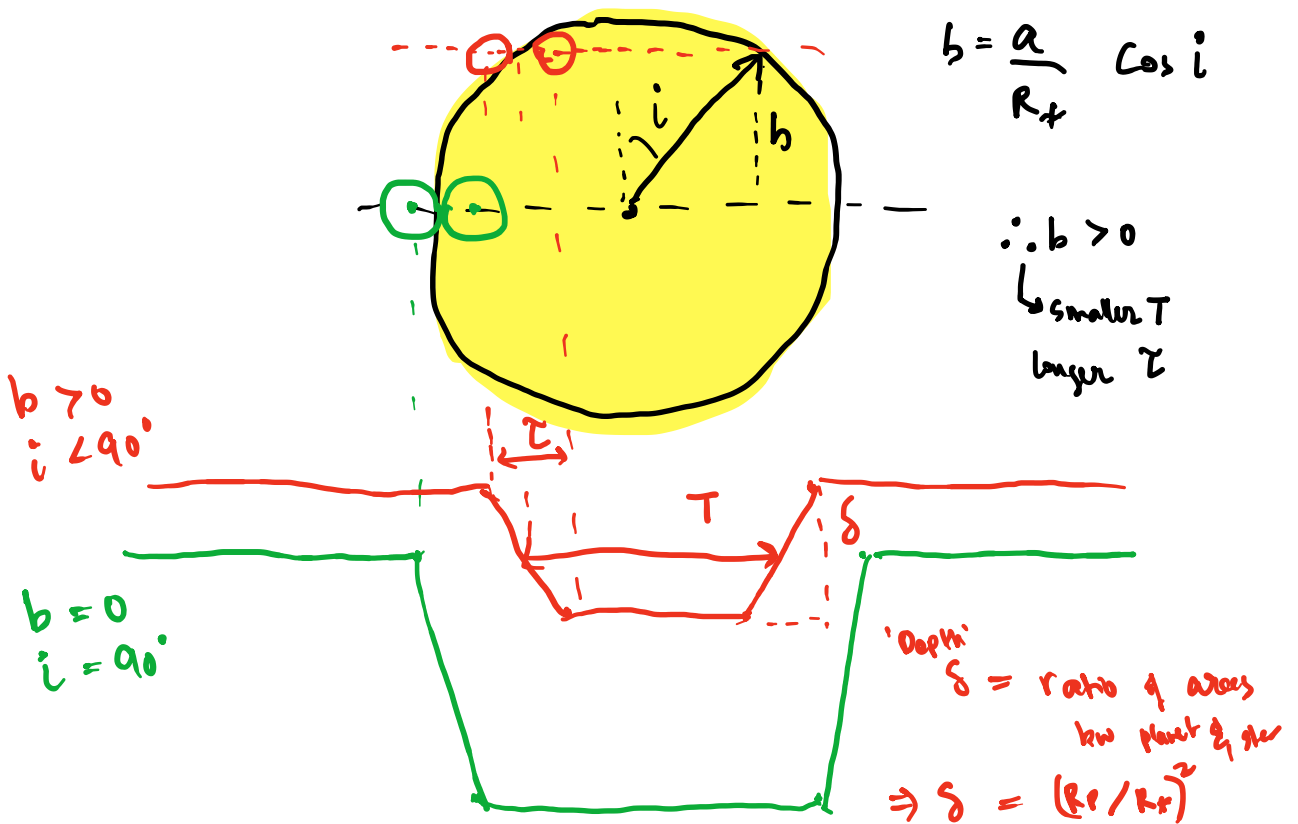
$$b = \frac{a \cos i}{R_*} \left(\frac{1 - e^2}{1 + e \sin \omega} \right)$$

$\downarrow R_* \ll a$

$$X = \pm R_* \sqrt{1 - b^2}$$
$$Y = b R_*$$



Assume not eccentric



NOTE:

1. Cant get R_{planet} directly from light curve -- you also need R_{star}
2. Bigger planets --> bigger depth
3. Smaller star --> bigger depth

Special case $b=0$

① τ

$$\text{distance} = 2R_p \Rightarrow \tau = \frac{2R_p}{v_p}$$

$$v_p = v_p$$

$$\Rightarrow \tau = \frac{2R_p}{(2\pi a/p)}$$

$$\Rightarrow \tau = \frac{pR_p}{\pi a}$$

② T

$$\text{distance} = 2R_* \Rightarrow T = \frac{2R_*}{v_p}$$

$$v_p = v_p$$

$$\Rightarrow T = \frac{pR_*}{\pi a}$$

General case $b > 0$

recall

$$b = (a/R_*) \cos i$$

① τ

$$\text{distance} = \frac{2R_p}{\sqrt{1-b^2}}$$

$$\Rightarrow \tau = \frac{pR_p}{\pi a \sqrt{1-b^2}}$$

② T

$$\text{distance} = 2R_* \sqrt{1-b^2}$$

$$\Rightarrow T = \frac{pR_* \sqrt{1-b^2}}{\pi a}$$

TRANSLATE OBSERVABLES \rightarrow PHYSICAL PROP

$$\frac{R_p}{R_*} = \sqrt{\delta}$$

$$b^2 = 1 - \delta^{1/2} \frac{T}{\tau}$$

$$\frac{a}{R_*} = \frac{p \delta^{1/4}}{2\pi} \left(\frac{4}{T\tau} \right)^{1/2}$$

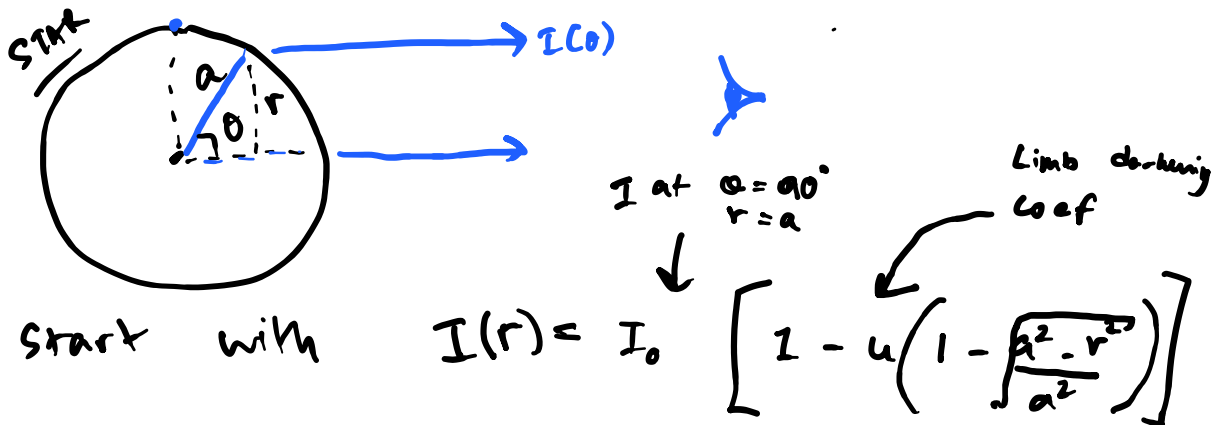
2. Limb Darkening

<https://arxiv.org/pdf/1308.0009.pdf>

<https://arxiv.org/pdf/astro-ph/0210099.pdf>

<https://arxiv.org/pdf/1901.01730.pdf>

<http://orca.phys.uvic.ca/~tatum/stellatm/atm6.pdf>



$$I(\theta) = I_0 \left[1 - u \cos \theta \right]$$

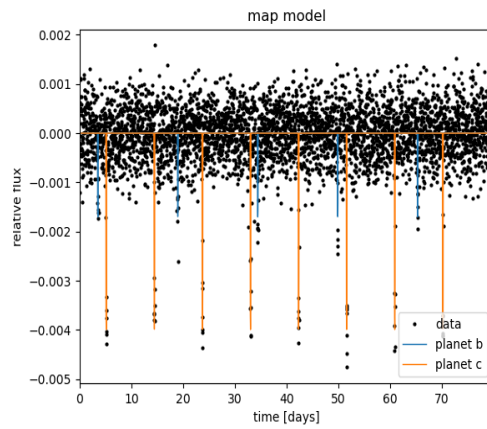
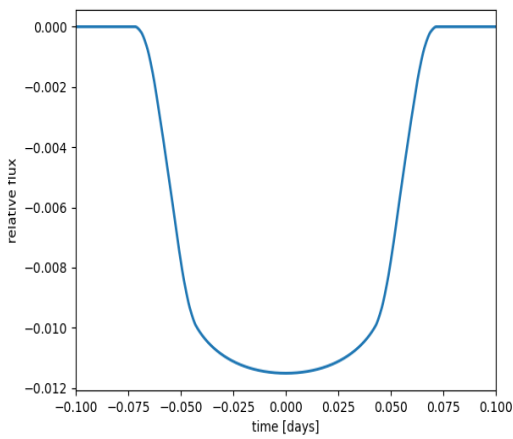
$$\Rightarrow u = \frac{I_0 - I(\theta)}{I_0 \sqrt{1 - r^2}}$$

Quadratic

$$\frac{I(\mu)}{I_0} = 1 - u_1 (1 - \mu) - u_2 (1 - \mu)^2$$

With limb darkening + Kepler's laws:

$$\Theta = \{P, t_0, \omega, e, i, a/R_*, R_p/R_*, v_c, u_{210}\}$$



3. Noise variability: Gaussian process with simple harmonic oscillator

